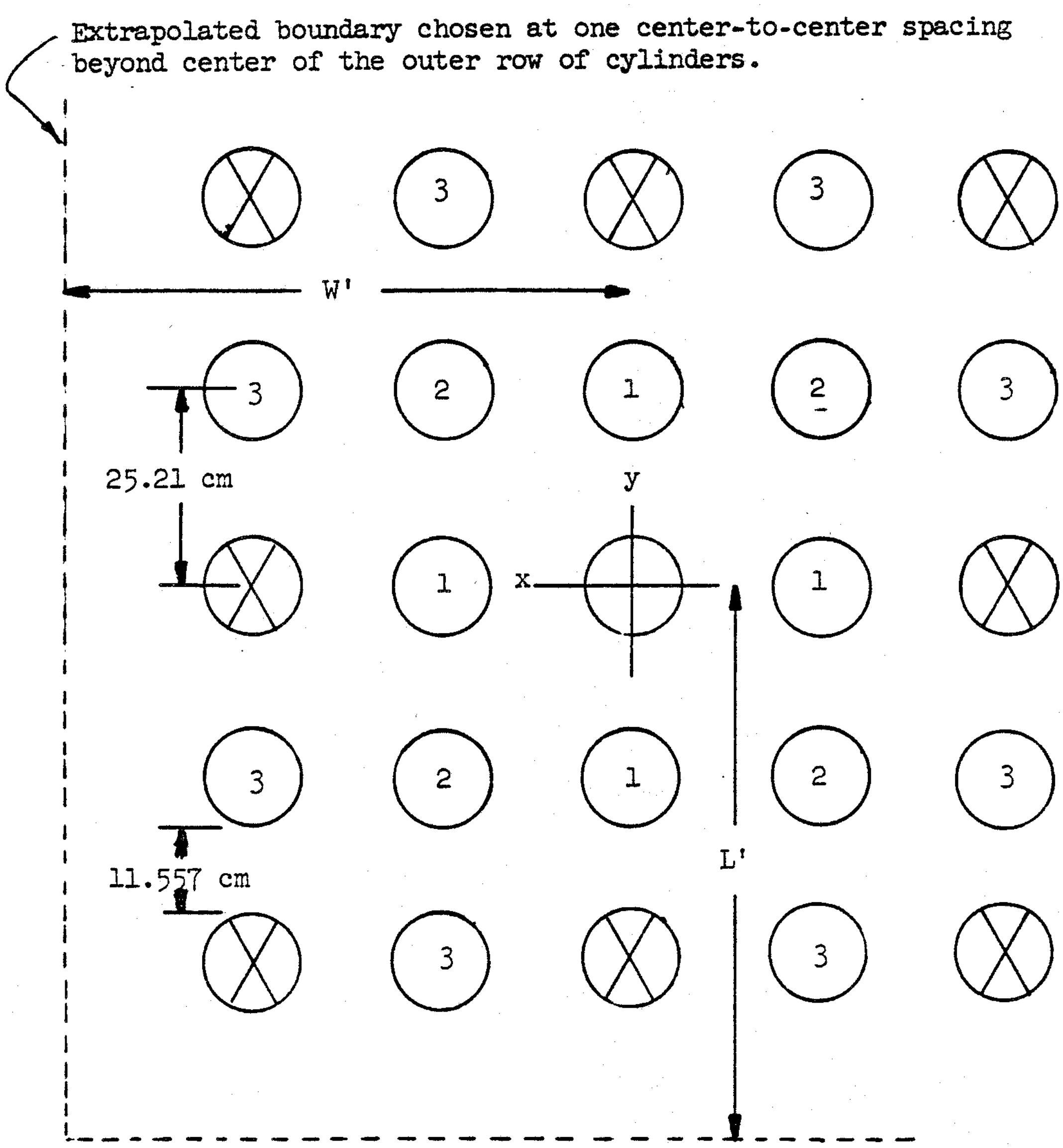
Example of Solid Angle Method and Correlation with Experiment

Problem: An array of 25 identical cylinders of 92.6 Wt% ^{235}U as uranyl nitrate at a concentration of 410 g U/l, the cylinders are polyethylene bottles 13.6525 cm O.D., 112.4 cm high, and average wall thickness of 0.63 cm. At critical the bottles are equally spaced at 11.557 cm surface to surface. Since the cylinders are identical the center-most will have the highest reactivity due to the interaction from all other "seen" cylinders. This was an actual critical experiment performed at Oak Ridge and reported in ORNL-3193, a progress report of the laboratory, in 1961.



From page III.B.11.93-1 we obtain k_∞ equal to 1.841 and M^2 equal to 28.7 and from page III.B.10.93-1 we find the bare extrapolation distance, λ_b, equal to 2.1.

The cylinders crossed out in the sketch, page V.B.2-5, are hidden from the center cylinder and do not interact with it. To obtain the fractional solid angle of each of the symmetry types, the center-to-center distance of each symmetry type from the central cylinder must be calculated.

Using the equation (b), the distance, h, must be obtained and the fractional solid angle calculated. For the closest cylinders to the central cylinder (symmetry one):

$$h = (\text{center-to-center distance}) - (\text{radius of the cylinder}) = 18.384 \text{ cm}$$

Then

$$\Omega_{fi} = \frac{.07962\ (13.6525)(112.4)}{18.384\left[(18.384)^2 + \left(\frac{112.4}{2}\right)^2\right]^{1/2}} = .1124$$

Since the array is planar and square, equation 3 in Table II may be used to calculate p. And since the array is in air

$$q = p = \cos\frac{\pi}{2}\left(\frac{x}{W'}\right)\cos\frac{\pi}{2}\left(\frac{y}{L'}\right) \quad \text{or}$$

$$q = \cos\frac{\pi}{2}\left(\frac{1}{3}\right)\cos\frac{\pi}{2}(0)$$

$$= 0.866$$

Since there are four cylinders of this symmetry, $\sum \Omega_{fi} q_i$ is equal to 4(0.866)(.1124) or 0.3893, the total solid angle for cylinders of symmetry one.

The solid angles for the other two symmetries are calculated in the same manner and are included in Table III, page V.B.2-8.

A second method for obtaining the fractional solid angle uses the curves on page V.D.1-2 to obtain values of λ and σ

where $\lambda = \frac{L}{d} = \frac{112.4}{13.6525} = 8.23$

and $\sigma = \frac{(\text{center-to-center distance}) - d}{d} = \frac{25.21 - 13.6525}{13.6525} = 0.85$

Applying these values to Figure V.D.1-3 gives Ω_f equal to 0.08.

The total solid angle for the four symmetry one cylinders, $\sum \Omega_{fi} q_i$, is then equal to 4(.866)(.08) or .277. The solid angle for the other symmetries are shown in Table III (p. V.B.2-8), which compares the solid angles calculated using equation (a) and using Figure V.D.1-3.

To calculate the k_{eff} of the array using equation (g), the geometric buckling, B_g^2, of a single unit must be calculated.

To calculate B_g^2 for one cylinder:

$$B_g^2 = \frac{J_0^2}{(R_{cy} + \lambda)^2} + \frac{\pi^2}{(H_{cy} + 2\lambda)^2} = \frac{5.784}{(6.8265 + 2.1)^2} + \frac{9.87}{(112.4 + 5.47)^2}$$

$$= .073298 \text{ cm}^{-2}$$

Note: Since the wall thickness of the polyethylene bottles varies, the outside dimension is used to allow for reflector savings. Reflector savings of 1.27 cm are added to the axial extrapolation distance.

Calculate k_a using equation (g):

$$k_a = \frac{k_\infty}{1 + M^2 B_g^2 \left[1 - \sum (q_i \Omega_{fi})\right]}$$

$$= \frac{1.841}{1 + (28.7)(.073298)(1 - .704)}$$

$$= 1.1345$$

This is compared to the experimental k_a of 1.000 giving a conservative result. If the total solid angle obtained by using Figure V.D.1-3 is used, the k of the array would be 0.9132, a nonconservative result. The results of the solid angle calculations of other arrays in this experiment are shown in Table IV (p. V.B.2-8). An examination of these results show that the solid angles obtained by the curves of Figure V.D.1-3 are nonconservative when used for close arrays as in this experiment, while the solid angle calculated using equation (b) yields a quite conservative, but safe, result. Therefore, use of the curves in Figures V.D.1-1, -2, and -3 should be limited to estimations of arrays of units that are separated by about two diameters or more.

Table IV also includes the k_{eff} calculated by computer codes Interset[4] and GEM-III,[15] and the calculated critical number of containers using the density analogue method for some of these arrays. Density analogue also yields nonconservative results for this array of tall, small diameter cylinders. Note also that Interset gives very nonconservative results.